Following Anderson et al.,[2] Eq. (20) is transformed by a local rotation matrix T so that the flux vector $\bar{F}$ can be split in a one-dimensional fashion in a direction along a $\xi = \text{constant}$ line, treating the $\partial\bar{G}/\partial\eta$ and $\partial\bar{H}/\partial\zeta$ as source terms. It is shown in Ref. 2 that the transformed flux vector

$$\hat{F} = T\bar{F}$$

is of the same form as the Cartesian flux vector and can thus be split according to the scheme developed for Cartesian coordinates after replacing the Cartesian velocity components by the corresponding rotated velocity components. The inverse transformation is then applied to the split fluxes $\hat{F}^{\pm}$ to obtain the splitting for $\bar{F}$. The split fluxes $\bar{F}^{\pm}$ are

$$\bar{F}^{\pm} = \frac{|\nabla\xi|}{J}\begin{pmatrix} \bar{F}_m^{\pm} \\ \dfrac{a}{\gamma}\bar{F}_m^{\pm}\left[\dfrac{\xi_x}{|\nabla\xi|}(-\bar{M}_\xi \pm 2) + \gamma M_x\right] \\ \dfrac{a}{\gamma}\bar{F}_m^{\pm}\left[\dfrac{\xi_y}{|\nabla\xi|}(-\bar{M}_\xi \pm 2) + \gamma M_y\right] \\ \dfrac{a}{\gamma}\bar{F}_m^{\pm}\left[\dfrac{\xi_z}{|\nabla\xi|}(-\bar{M}_\xi \pm 2) + \gamma M_z\right] \\ \bar{F}_E^{\pm} \end{pmatrix} \tag{21}$$

where

$$\bar{F}_m^{\pm} = \pm\frac{\rho a}{4}(\bar{M}_\xi \pm 1)^2$$

$$\bar{F}_E^{\pm} = \frac{\gamma^2}{2(\gamma^2-1)}\left[\frac{(\bar{F}_{M_\xi}^{\pm})^2}{\bar{F}_m^{\pm}} + \frac{2}{\gamma^2}\frac{\xi_t}{|\nabla\xi|}\bar{F}_{M_\xi}^{\pm} + \frac{1}{\gamma^2}\frac{\xi_t^2}{|\nabla\xi|^2}\bar{F}_m^{\pm}\right] + \frac{\bar{F}_m^{\pm}}{2}(\bar{v}^2 + \bar{w}^2)$$

$$\bar{F}_{M_\xi}^{\pm} = \frac{a}{\gamma}\bar{F}_m^{\pm}[(-\bar{M}_\xi \pm 2) + \gamma M_\xi]$$

$$a\bar{M}_\xi = (u\xi_x + v\xi_y + w\xi_z + \xi_t)/|\nabla\xi| = \bar{u} + \xi_t/|\nabla\xi|$$

$$aM_\xi = \bar{u}$$

Note also that

$$u^2 + v^2 + w^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2$$

where $\bar{u}$, $\bar{v}$, and $\bar{w}$ are the rotated velocity components. The experession corresponding to Eq. (21) for stationary curvilinear coordinates is presented by Thomas et al.[3] The split fluxes $\bar{G}^{\pm}$ and $\bar{H}^{\pm}$ are obtained from Eq. (21) by replacing ξ by η and ζ, respectively.

Concluding Remarks

In this Note, the van Leer split-flux vectors have been derived for moving curvilinear coordinate systems. The split fluxes presented reduce to those given by van Leer[1] and Thomas et al.[3] for the case of stationary coordinates.

The split-flux vectors obtained in the present study have been successfully applied to a fixed-wing calculation in which the relative motion between the wing and the fluid was introduced through the grid motion. The application of the result obtained here to the calculation of helicopter rotor flowfields is currently being investigated.

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Turbulent Near Wake of a Symmetrical Body

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Nomenclature

C_a	= Crocco number of adjacent flow
k	= ratio of specific heats
M_a	= Mach number of adjacent flow
u	= velocity
u_c	= centerline velocity
u_a	= velocity adjacent to wake region
u_{max}	= maximum value of adjacent velocity if all the energy of the flow is in the form of kinetic energy
r	= radius
R	= radius where adjacent flow begins
y	= two-dimensional coordinate normal to flow direction
Y	= y where adjacent flow begins
ρ	= density
ρ_a	= adjacent flow density

Introduction

WHEN a symmetrical body is immersed in a uniform flow at zero angle of attack, an interesting conclusion can be drawn about the near wake profile. The conclusion is limited to those wakes in which vortex shedding is not present or is negligible.

Formulation

The location of the near wake profile is defined to be downstream of the body where the pressure gradient normal to the centerline has vanished and the time averaged streamlines have become straight and parallel. If we assume that dominant vortex shedding is not present, the time averaged values of the velocity in the near wake profile can be approximately represented by a cosine function as shown in Fig. 1. Thus, in the near wake

$$\frac{u}{u_a} = \frac{u_c}{u_a} + (0.5)\left(1 - \frac{u_c}{u_a}\right)\left(1 - \cos\left[\frac{r}{R}\,180\text{ deg}\right]\right) \tag{1}$$

where $r \to y$ and $R \to Y$ for the two-dimensional case.

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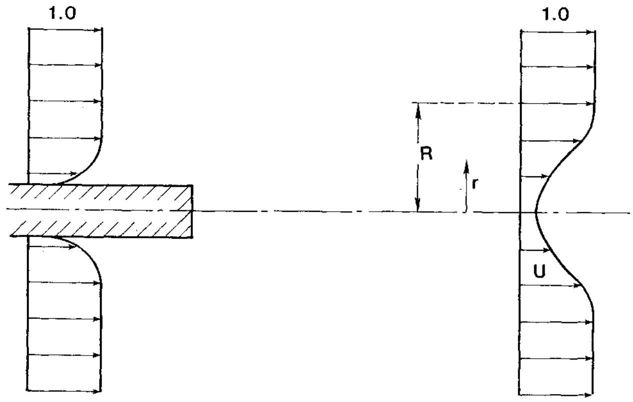

Fig. 1 Dimensionless velocity profiles in constant pressure regions.

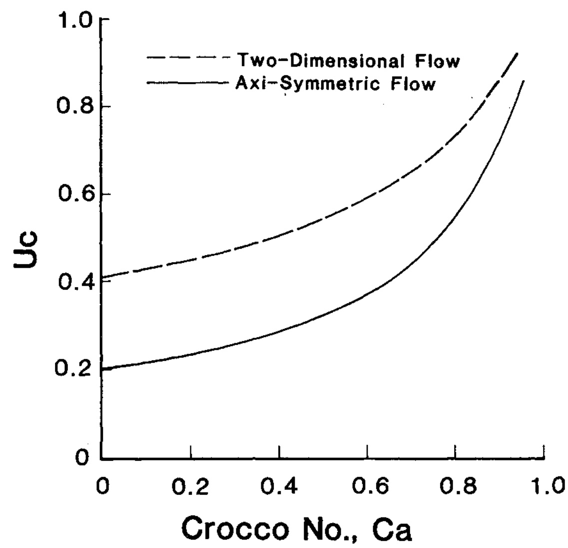

Fig. 2 Dimensionless centerline velocity vs Crocco number.

One version of the unified principle of thermodynamics[1] or the Second Law of Thermodynamics[2] is that the final equilibrium state of an isolated system is the state with maximum entropy. This principle can be applied to the near wake and has been useful in selecting one possible equilibrium state from a large number of choices in solving axisymmetrical wake problems.[3-5]

If a control volume is placed around the flow shown in Fig. 1, such that the outer control surface is in a constant pressure region parallel to the centerline, the dimensionless entropy flux from the control volume can be evaluated for a perfect gas at the constant pressure present in the control surface. The dimensionless entropy flux is represented by Eq. (2) for the axisymmetrical case where the flow is turbulent and the Reynolds number is very large:

$$\Sigma_{AS} = \int_0^1 \frac{\rho}{\rho_a} \frac{u}{u_a} \ln\left(\frac{\rho_a}{\rho}\right)\left(\frac{r}{R}\right) \mathrm{d}\left(\frac{r}{R}\right) \tag{2}$$

Here, ρ is made dimensionless by the undisturbed adjacent density.

For the two dimensional case without dominant vortex shedding, the dimensionless entropy flux is shown by Eq. (3). Again, this is restricted to a case of turbulent flow with high Reynolds number in order that the effects of body boundary layer are negligible:

$$\Sigma_{2D} = \int_0^1 \frac{\rho}{\rho_a} \frac{u}{u_a} \ln\left(\frac{\rho_a}{\rho}\right) \mathrm{d}\left(\frac{y}{Y}\right) \tag{3}$$

If one uses Eq. (1) for the near wake veloicty profile and a turbulent Prandtl number of unity in order that Eq. (4) can represent the density profile,[6] then the values of the near wake centerline velocity which maximizes Eqs. (2) and (3) can be determined. The results of these calculations are shown in Fig. 2.

$$\frac{\rho}{\rho_a} = \frac{1 - C_a^2}{1 - (u^2/u_a^2)C_a^2} \tag{4}$$

where

$$C_a^2 = (u_a^2)/(u_{\max}^2) = M_a^2/[M_a^2 + (2/k-1)] \tag{5}$$

Conclusion

Figure 2 presents the centerline velocity values which provide the most probable equilibrium solution. If one chooses to use a different but appropriate model for the near wake velocity profile or to account for Prandtl numbers different than unity, one will obtain slightly different numerical results. But, the important conclusion of the Note is that there exists a unique value of the time averaged centerline velocity in the near wake of a symmetrical body as a limiting case for turbulent high Reynolds number flow.

Acknowledgment

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Shock Wave Formation in a Suddenly Compressed Rubber Rod

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